ISSN: 2348 - 2117

International Journal of Engineering Technology and Computer Research (IJETCR)

Available Online at www.ijetcr.org

Volume 11; Issue 02;March-April; 2023; Page No. 01-06

To Approximate the Polar Derivative of a Polynomial with Coefficients, one may use

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Conflicts of interest: Nil

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Abstract

In this work, we look at the polynomial class $\rho(z)=b_0+\sum_{j=\mu}^{n} b_j z^j, 1\leq$ μ≤ no f degree n, having all zero son |z|=k,k≤1,Novel inequalities are established between the uniform norm of a polynomial and its polar derivative under the condition that the number of zeros in the polynomial is constrained. Several previously demonstrated inequalities for restricted polynomials are improved upon by these findings, and new, sharper inequalities are generated, adding to the already extensive body of research on the topic. In particular situations, our theory also provides some interesting extensions of certain Zygmund type inequalities for polynomials. This research adds to the expanding body of knowledge on inequalities that link A team of researchers from the United Kingdom and the United States conducted this study. When a polynomial's zeros are limited, we create some additional requirements that link the polynomial's uniform-norm to its polar derivative. These restrictions are only true when the polynomial's zeros are limited. The findings are consistent with the newly discovered Erd osLax and Tur'an inequalities for restricted polynomials. Moreover, the revelations establish a number of disparities that are more pronounced than those previously recognized in the vast amount of study on this issue

Keywords: Zeros, polar derivatives, and complex domains

Introduction

$$If\ \rho(z) = \sum_{j=0}^{n} bjzj$$

In the complex plane, stands in for n, or its derivative J(z) may be written as. In addition, Bernstein is responsible for a well recognised injustice [2]:

$$\max_{|za|=1} |\rho^{J}(za)| \leq n \underset{|za|=1}{\text{maximum}} |\rho(za)| \quad (1)$$

These maximum module principle, and straightforward conclusion from that principle ([20],

$$\underset{|z|=\zeta>1}{\text{Maximum}} |\rho(z)| \leq \zeta^{n} \max_{|za|=1} |\rho(z)|. \quad (2)$$

Both (1) and (2) are equivalently true if and only if (z) has a unique solution with all zeros at the origin. All result in variations and generalisations of the aforementioned inequality (1.1), which was developed by Bernstein in 1912. One way to emphasise the above inequality (1.1) is to restrict consideration to polynomials with z 1. In fact, Erd os

$$\max_{|z|=1} |\rho^{J}(za)| \leq \frac{n}{2} \max_{|z|=1} |\rho(za)|. \quad || \quad (3)$$

For all real values of z, the best possible inequality is (3), has no nonzero values except for z = 1.

Expanding on the result in (3), Malik [11] showed that if (z) = 0 in z n, n> 1, then

$$\underset{|z|=1}{\text{maximum}} |\rho^{J}(za)| \leq \frac{an}{1+k} \underset{|z|=1}{\text{maximum}} |\rho(za)|. \quad (4)$$

These sharp result has the extreme polynomial as (za + k)n, which is a very small number.

Govil [4] demonstrated then we get an inequality equivalent to (1.4) for non-vanishing polynomials in |z| n, n 1.

$$\underset{|z|=1}{\text{maximum}} |\rho^{J}(za)| \leq \frac{n}{k^{n-1}+k^{n}} \underset{|z|=1}{\text{maximum}} |\rho(z)|. \quad (5)$$

The set of polynomials (z) = b0 +n j=µthe possession of bjzj, 1 n

all 0's on |z| = k, k 1, an extension by Khojastehnezhad and Bidkham [9] inequality (1.5), and shown that if (z) = b0 + c, then (z) = c.

j=µbjzj, 1 ≤ µ ≤ n

$$\underset{|z|=1}{\text{maximum}} |\rho^{J}(z)| \leq \frac{n}{k^{n+\mu-1}+k^{n+\mu+2}} \underset{|z|=1}{\text{maximum}}(za)|. \quad (6)$$

Let us first recall that the definition of the polar derivatives D(za) of (z) with a regard as follows:

$$D_{\gamma}\rho(z):=n\ \rho(za)+(\gamma-z)\rho^{J}(za).$$

This is what is known as, calculated with respect to, and it is important to with respect to z. (see [12], [13] or [19]). Since it generalizes the concept of an ordinary derivative,

$$\lim_{\gamma\to\infty}\frac{D_{\gamma}\rho(z)}{\gamma}:=\rho^{J}(z),$$

R > 0 holds true consistently with regard to z.

Other authors have produced a huge variety of versions and generalisations of the aforementioned inequalities, of biggest roots of (za), limitation, use of by placing constraints on the multiplicity of zeros. Some of these oversimplifications involve contrasting values of the polar, and other parameters. The articles include the most up-to-date findings from research on this issue ([6], [7], [14]-[18]).

The major purpose of this study is to provide upper limit maximum The answers provided herein both refine and generalise several established approximations for the ordinary derivative of polynomials.

2. Main Result's

Theorem A. Lets $\rho(z)=b_0+\sum_{=\mu} b_j z^J, 1\leq\mu\leq n\ a$

is an integer-degree polynomialn, where |za| = n, k 1, is all zeros, n, we are at

$$\max_{|z|=1}|D_{\gamma}\rho(z)|\leq\frac{n(|\gamma|+k^{\mu})}{k^{n+\mu-1}+k^{n+\mu-2}}\max|\rho(z)|.$$

$$||\qquad\to\infty\qquad(2.1)$$

Inequality (2.1) may be solved by stand for a constant (1.6). Thereafter, we may improve upon this constraint by using the polynomial's extremal coefficients (2.1). We provide a more in-depth proof following outcomes.

Theorem b. Let $\rho(z)=b_0+\sum_{j=\mu}^{n} b_j z^J, 1\leq\mu a\ \leq n\ a$

$$\underset{|z|=1}{\text{maxi mum}}\ |D_{\gamma}\rho(z)|\leq\frac{n(|\gamma|+\psi(\mu,k))}{k^{n+\mu-1}+k^{n+\mu-2}}\underset{|z|=1}{\text{maximum}}\ |\rho(za)|,\qquad(2.2)$$

We proves that these limit in (2.2) is superior than the one in (2.1) by demonstrating that

$$\psi(\mu,k)=\frac{n|Z_n|ka^{2\mu}+\mu|b_{n-\mu}|ka^{\mu a-1}}{n|b_n|k^{\mu-1}+\mu|b_{n-\mu}|}\leq k^{\mu}$$

$$n|b_n|k^{2\mu}+\mu|b_{n-\mu}|k^{\mu-1}\leq n|b_n|k^{2\mu-1}+\mu|b_{n-\mu}|k^{\mu},$$

which implies or

$$n|b_n|(k^{2a}-k^{2\mu-1})\leq\mu|b_{na-\mu}|(k^{\mu}-k^{\mu-1})Z\underline{b_n}$$

$$\mu b_{n-\mu}\geq\frac{1}{k^{\mu}},$$

Remark1. Inequality (2.2) may be derived by setting = 1 and dividing each side by ||, then setting || . (1.5).

Result 1 Let (z) =nj=0bjzj is an n-degree polynomial that ends in zero on all its roots if we fix |z| = kn, n 1, then for complex numbers with k, we have

$$\psi(1,k)=\frac{n|b_n|k^2+|b_{n-1}|}{n|b_n|+|b_{n-1}|}. \tag{2.5}$$

Remark2. For μ = n, If the b0 k bn, then Theorem 2 is true. As a result of the theory that follows, the aforementioned inequality (2.4) will follow as a outcome.

Theorem3. Let$\rho(z)=\sum^{n} b_j z^j$ *for any we obtain, where is a polynomial of degree n with all zeros on |z| = n, kn 1.*

$$\max_{|z|=}|D_v\rho(z)|\leq\frac{}{k^n+R\,k^{\,n-1}}\;\frac{}{r^n+k\,r^{\,n-1}}\max_{|z|=r}|p(z),| \tag{2.6}$$

Remark3.Inequality (2.6) is resolved to its negation if we set R = r = 1. (2.4). The following extension of Theorem 3 is established by the use of (z) = bjzj, j=0.

Lemma1.Let $\rho(z)=b_n z^n+\sum_{j=\mu}^{n} b_{n-j}z^{n-j}, 1\leq\mu a\leq na,$ is a polynomial of

where s $\rho^*(z)=z^n\rho\left(\frac{1}{\bar{z}}\right)$.

Lemma2. Let $\rho(z)=b_n z^n + \sum_{j=\mu}^{n} b_{n-j} z^{n-j}, 1\leq\mu\leq n$, is a polynomials of degree, having all zero son $|za|=kn, kn\leq 1$, thenfor$|za|=1$,

$$\psi(\mu, k)|\rho^{J}(z)|\geq|(\rho^{*}(z))^{J}|$$

3. Proof s of theorem

Proof of Theorem 1. Then $\rho(za)=b_0 \sum_{j=\mu}^{n} b_j z^j, 1\leq\mu<n$ is a polynomial of degree ,having all zero son $|za|=n, n\leq 1$.Let$\rho^{*}(za)=z^n\rho(\frac{1}{})$,

can easily substantiated that $(\rho^{*}|(z)|)^{J}=n\ \rho(z)\ z\rho^{J}(z)$ for $|z|=1$.

$$\begin{aligned} |D_{\gamma}\rho(z)| &= |n\ \rho(z)+(\gamma-z)\rho^{J}(z)| \\ &= |\gamma||\rho^{J}(z)| + |(\rho^{*}(z))^{J}| \\ &\leq |\gamma|+\psi(\mu,k)|\rho^{J}(z)|. \end{aligned} \tag{4.1}$$

$$|\rho^{J}(z_0)|+\max_{|z|=1}|(\rho^{*}(z))^{J}|\leq n\ \max_{|z|=1}|\rho(z)|,$$

which s is used Lemma 5 yield,

$$\frac{1}{k}|(\rho^{*}(z_0))^{J}|+|(\rho^{*}(z_0))^{J}|\leq n\ \max_{|z|=1}|\rho(z)|,$$

$$\frac{1}{k}+1\max|(\rho^{*}(z))^{J}|\leq n_k \max_{|z|=1}|\rho(z)|. \qquad \max_{|z|=1}|\rho(z)|. \tag{4.2}$$

$$\underset{|z|=1}{\text{maximum}}|\rho^{J}(za)|\leq\frac{n}{k^{n+\mu-1}+k^{n+\mu-2}} \quad {}_{|z|=1}$$

On com binning in equalities (4.1a) and (4.2b),we get

$$\underset{|z|=1}{\text{maximum}}\ |D_{\gamma}\ \rho(z)|\leq\frac{k(|\gamma|+\psi(\mu,k))}{k^{n+\mu-1}+k^{n+\mu-2}}\ \underset{|z|=1}{\text{maxi mum}}\ |\rho(za)|.$$

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